STUDY OF TWO-LAYER FLOW OF VISCOELASTIC MOLTEN POLYMERS IN AXISYMMETRICAL CHANNELS

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One of the stages of producing multilayer films and two-component fibers (including optical fibers) is joint extrusion of two or more molten polymers through a forming channel. Here there is layered or stratified flow [1]. Areas of steady-state stratified flow are the most completely studied where the profile of the flow rate and the position of the interface of the liquid layers is unchanged over the length of the channel [1-4]. Fewer works are devoted to studying the initial area of combined flow where, as is shown in an experiment in [5] and in theoretical finite element analysis [6], the coordinates of the interface vary considerably.

In this work results are offered for a numerical study of two-layer flow of immiscible viscoelastic liquids. A mathematical flow model and a method for numerical realization are provided, and the results of calculations are analyzed.

1. Statement of the Problem and Method of Solution. Stationary axisymmetrical two-layer flow of viscoelastic liquids of equal densities in a cylindrical channel is considered. Variables are the vortex ω and flow function ψ, which are connected with velocity vector components v_r and v_z so that

$$\omega = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}, \quad v_z = \frac{1}{r\rho}\frac{\partial \psi}{\partial r}, \quad v_r = -\frac{1}{r\rho}\frac{\partial \psi}{\partial z},$$

and differential equations which determine the mathematical flow model have the form [7]

$$\left[\frac{\partial}{\partial z}\left(\frac{\omega}{r}\frac{\partial \psi}{\partial r}\right) - \frac{\partial}{\partial r}\left(\frac{\omega}{r}\frac{\partial \psi}{\partial z}\right)\right] r^2 - \tag{1.1}$$

$$- \frac{\partial}{\partial z}\left(r^3 \frac{\partial}{\partial z}\left(\mu \frac{\omega}{r}\right)\right) - \frac{\partial}{\partial r}\left(r^3 \frac{\partial}{\partial r}\left(\mu \frac{\omega}{r}\right)\right) + S_w r^2 = 0;$$

$$\frac{\partial}{\partial z}\left(\frac{1}{r\rho}\frac{\partial \psi}{\partial z}\right) + \frac{\partial}{\partial r}\left(\frac{1}{r\rho}\frac{\partial \psi}{\partial r}\right) + \omega = 0, \tag{1.2}$$

where r and z are coordinates; ρ is density; μ is polymer viscosity.

In equations for vortex strength (1.1) the source term is determined by a selected rheological equation of state (RES). We use the de Witt model as an RES which predicts an anomaly for liquid viscosity and also appearance in steady-state flow of first and second differences for normal stresses [1]:

$$\tau_{ij} + \lambda D_J \tau_{ij}/(D_J t) = \mu d_{ij}. \tag{1.3}$$

Here λ is liquid relaxation time; τ_{ij}, d_{ij} are stress deviator tensor components and strain rate tensor; $D_J/D_J t$ is Yauman derivative. Taking account of (1.3) the expression for S_w is written as

$$S_w = -2\left[\frac{\partial^2 \mu}{\partial r\,\partial z}\left(\frac{\partial v_r}{\partial r} - \frac{\partial v_z}{\partial z}\right) + \frac{\partial^2 \mu}{\partial z^2}\frac{\partial v_z}{\partial r} - \frac{\partial^2 \mu}{\partial r^2}\frac{\partial v_r}{\partial z}\right] + \frac{\partial^2}{\partial r\,\partial z}(\lambda u_1) +$$

$$+ \left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial r^2}\right)(\lambda u_2) - \frac{1}{r}\frac{\partial}{\partial z}(\lambda u_3) - \frac{1}{r}\frac{\partial}{\partial r}(\lambda u_2) + \frac{1}{r^2}\lambda u_2,$$

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where

$$u_1 = \frac{\partial}{\partial r}(v_r(\tau_{rr} - \tau_{zz})) + \frac{\partial}{\partial z}(v_z(\tau_{rr} - \tau_{zz})) + \frac{v_r}{r}(\tau_{rr} - \tau_{zz}) - 2\omega\tau_{rz},$$

$$u_2 = \frac{\partial}{\partial z}(v_z\tau_{rz}) + \frac{\partial}{\partial r}(v_r\tau_{rz}) + \frac{v_r}{r}\tau_{rz} + \frac{1}{2}\omega(\tau_{rr} - \tau_{zz}),$$

$$u_3 = \frac{\partial}{\partial z}(v_z(\tau_{\theta\theta} - \tau_{rr})) + \frac{\partial}{\partial r}(v_r(\tau_{\theta\theta} - \tau_{rr})) + \frac{v_r}{r}(\tau_{\theta\theta} - \tau_{rr}) + \omega\tau_{rz}.$$

Equations (1.1)-(1.3) will be correct for both liquids with consideration of μ, λ, and the position of a liquid with respect to the interface F. In implicit form the equation for the interface has the form

$$F(r, z) = 0. \tag{1.4}$$

Mathematical formulation of this equation is refined after considering the boundary conditions of the problem.

We shall assume that at the inlet to the zone of combined flow individual flows are hydrodynamically developed and at the outlet from the channel conditions of steady-state two-layer flow are fulfilled. Also conditions of axisymmetrical flow, impenetrability of the channel walls, and sticking of liquids to them are fulfilled.

At the interface equality is assumed for the velocities of touching liquid particles and there is also equality for stresses which operate from the direction of the first to the second liquid (the effect of surface tension is ignored in view of its smallness) [8]:

$$\mathbf{v}^a = \mathbf{v}^b, \ \mathbf{r} \in F; \tag{1.5}$$

$$\mathbf{S}_n^a = \mathbf{S}_n^b, \quad \mathbf{r} \in F \tag{1.6}$$

($\mathbf{v}^a$ and $\mathbf{v}^b$ are velocity vectors, and $\mathbf{S}_n^a$ and $\mathbf{S}_n^b$ are the vectors of stresses at the liquid interface orientated with normal $\mathbf{n}$).

Condition (1.6) may be written in terms of stress tensor components in projections on normal $\boldsymbol{n}$ and tangent $\mathbf{t}$ to the interface:

$$[(-p^a\delta_{ij} + \tau_{ij}^b)n_i]\,n_i^{\mathrm{T}} = [(-p^b\delta_{ij} + \tau_{ij}^b)n_i]\,n_i^{\mathrm{T}}; \tag{1.7}$$

$$[(-p^a\delta_{ij} + \tau_{ij}^a)n_i]\,t_i^{\mathrm{T}} = [(-p^b\delta_{ij} + \tau_{ij}^b)n_i]\,t_i^{\mathrm{T}}. \tag{1.8}$$

Here δ_{ij} is Kronecker symbol; p^a, p^b are hydraulic pressures in the layers; n_i^T, t_i^T are transposed matrices of the vectors.

It follows from condition (1.5) that the interface is a flow surface. The value of flow functions in it are determined from the condition for constancy of the flow rates of both liquids Q^a and Q^b:

$$Q^a = 2\pi \int_0^{R_z(z)} v_z^a r\,\partial r; \tag{1.9}$$

$$Q^b = 2\pi \int_{R_z(z)}^{R_2} v_z^b r\,\partial r \tag{1.10}$$

[$R_z(z)$ are coordinates of the interface, R_2 is channel radius].

It is possible to assume that at the channel axis the flow function equals zero, and at the interface its values are constant, i.e., $\psi^a(R_z(z), z) = \psi^b(R_z(z), z)$. Then from conditions (1.9) and (1.10) we obtain values of flow functions:

at the interface

$$\psi(R_z(z), z) = \rho Q^a/2\pi; \tag{1.11}$$

at the channel wall

$$\psi(R_2, z) = \rho(Q^a + Q^b)/2\pi. \tag{1.12}$$

Whence it follows that the phase interface is determined by condition (1.11) which is thus an equation for finding the coordinates of the liquid interface $R_z(z)$ in two-layer axisymmetrical flow.

Boundary conditions for the vortex $\omega^{a,b}(R_z(z), z)$ may be obtained by laying out in a series the flow function in the direction of the normal to the interface and taking account of relationships (1.5), (1.6), (1.8). However, it is very difficult to realize these conditions.

Therefore, we use an approximate method with smoothed coefficients [9]. In essence this method consists of the fact that at the interface a phase is not clearly separated since separation functions in it are substituted by continuous functions within the limits of the small vicinity of this boundary. Here it is not required to state boundary conditions at the interface (1.5) and (1.6) since they are fulfilled automatically in view of the continuity introduced for the functions.

The possibility of applying this method for studying processes of stratified flow of liquids is physically sound for the diffusion of molten polymers with combined flow. Therefore, it is logical to assume that within the limits of the vicinity of the interface viscosity and relaxation time vary from values in the inner layer to those in the outer layer. Smoothing of functions μ and λ should be continuous and have continuous derivatives. Therefore, for conjugation we use polynomials, i.e., third or fifth order parabola. Thus, the distribution of viscosity will be as follows:

$$\mu(r, z) = \begin{cases} \mu^a(r, z), & r \leqslant R_z(z) - \delta, \\ \mu^0(r, z), & R_z(z) - \delta \leqslant r \leqslant R_z(z) + \delta, \\ \mu^b(r, z), & r \geqslant R_z(z) + \delta \end{cases} \tag{1.13}$$

[$\mu^0(r, z)$ and δ are viscosity and half-thickness of the boundary layer]. The distribution rule for relaxation time is also similar. It is noted that since coordinates $R_z(z)$ are for values of flow function (1.11), then smoothing is actually carried out with respect to ψ, and interval δ corresponds to a specific range of values of ψ. By solving the boundary problem for equations with smoothed coefficients we obtain an approximate solution for the original boundary problem which will be more accurate, the less the width of the smoothing zone. In solving the problem by the grid method interval 2δ should include along axis r not less than five nodes of the calculation grid, and it is desirable that the pitch of the grid within this range is constant.

In order to obtain difference analogs of differential Eqs. (1.1) and (1.2) we use an integral method [7]. We solve the set of algebraic equations by the Liebman iteration method. Coordinates of the interface $R_z(z)$ are determined from condition (1.11) by additional iterations.

Numerical solution of this problem in the region of steady-state flow was compared with the analytical solution [2], and in the initial section the particular solution (for Newtonian liquids) was compared with results in [6]. The maximum divergence of results for the velocity field did not exceed 5%. Calculations were carried out in the range of Reynolds numbers $\mathrm{Re} = 10^{-2}$-10^{-5} and Weissenberg numbers $\mathrm{We} = 2\cdot10^{-1}$-$10^{-3}$ for both flows.

2. Analysis of the Results Obtained. Presented in Fig. 1 is the velocity field for two-layer flow of liquids with flow rates $Q^a = Q^b = 60\cdot10^{-9}$ m³/sec, viscosities $\mu^a = 4000$ Pa·sec, $\mu^b = 6300$ Pa·sec, and relaxation times $\lambda^a = \lambda^b = 10^{-3}$ sec in a channel with $R_2 = 3\cdot10^{-3}$ m. The radius of the channel from which the inner melt A enters with velocity $v_z(0, 0) = 6\cdot10^{-3}$ m/sec, $R_1 = 0.87R_2$. Lines 1-5 relate to $\psi\cdot10^6 = 0$, 1, 3.8, 9.55, 16.7 kg/sec. It is possible to separate two areas in the velocity field. The first in which there is rebuilding of the flow rate profile and where flow lines are not parallel to the channel wall, by analogy with single-layer flows we call an area of hydrodynamic stabilization (HDS). The second, where flow lines are parallel to each other and the wall, and the velocity profile is unchanged over the channel length, is an area of steady-state flow.

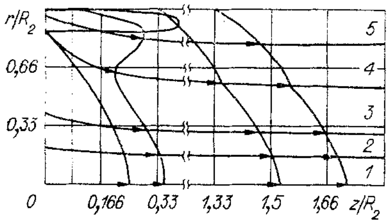

Fig. 1

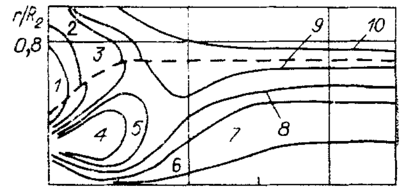

Fig. 2

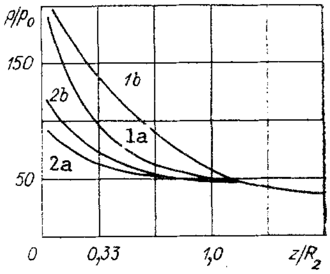

Fig. 3

By using Eq. (1.11) we determine the layer interface. It coincides with flow line $\psi = 9.55 \cdot 10^{-6}$ kg/sec. The liquid interface is not a continuation of the inner channel surface. Its coordinates vary along the length in a specific way up to a constant value in the zone of steady-state flow which only depends on the ratio of flow rates and the ratio of liquid viscosities at the interface [3]. The radius of the inner channel R_1 only determines the shape of the interface in the initial section.

At the inlet to the zone of combined liquid flow there are marked differences in velocity profiles. With merging of the flows there is acceleration of liquid boundary layer particles and simultaneously a change in the coordinates of the interface. Considerable deformations arise in two-layer flow. As the liquids advance over the channel deformations in the layers are leveled out and in the region of steady-state flow their difference is only determined by the rheological properties of the materials [3].

It seems that many defects of polymer layered articles (instability of the interface, disruption of adhesion between layers) are the result of the specific stress—strained state of flow in the initial section. We consider the distribution of tangential stresses in two-layer flow (Fig. 2). Flow parameters here are the same as for Fig. 1, only $Q^a = 60 \cdot 10^{-9}$ m^3/sec, $Q^b = 15 \cdot 10^{-9}$ m^3/sec, and $R_1 = 0.4R_2$. Curves 1-10 relate to $\tau_{rz} = 0$, 0.01, 0.0125, 0.03, 0.02, 0.005, 0.01, 0.012, 0.014, 0.016 MPa. In the initial section where stresses vary in the axial and radial directions values of τ_{rz} are discontinuous at the interface. However, the value of a jump is small, and as predicted by Eq. (1.8) it decreases along the channel length to zero in the region of steady-state flow. We note a marked zone of positive values of τ_{rz} bounded by curve 1 (Fig. 2), and also the high intensity of the change in stresses in the initial section.

For practical purposes it is important to know the length of the HDS section L. With flow of viscous liquids it is normally estimated from the change in the flow velocity profile or from the section of tangential stress relaxation. Calculations carried out for two-layer flows of Newtonian liquids showed that for values of Re flow of the order of 10^{-3}-10^{-6} L it is (1.0-2.0)R_2 in relation to radius R_1 at the inlet. With flow of viscoelastic liquids length L will be the size of the section of normal stress relaxation. For liquids described by the de Witt model with an increase in We, growth of L by 10-15% is observed compared with flows of Newtonian liquids.

The length of the HDS section may also be estimated from the distribution of pressure in two-layer flow. Presented in Fig. 3 are curves for the change in pressure at the interface of Newtonian liquids along the channel length in relation to inner channel radius R_1. Here $Q^a = 60 \cdot 10^{-9}$ m^3/sec, $Q^b = 30 \cdot 10^{-9}$ m^3/sec, $\mu^a = 4000$ Pa·sec, $\mu^b = 2000$ Pa·sec. Curves 1a, b correspond to pressures of $p^a(R_z(z), z)$ and $p^b(R_z(z), z)$ with $R_1 = 0.4R_2$, and curves

2a, b correspond to pressures with $R_1 = 0.65R_2$. As follows from boundary conditions (1.7) in the general case the pressure field is discontinuous at this interface. In the region of steady-state flow when the line of the layer interface is parallel to axis z and normal stresses in Newtonian liquids are zero, pressures p^a and p^b at the interface are equal. It can be seen from Fig. 3 that in the second case when coordinates $R_z(z)$ are almost unchanged along the channel length (for given ratios of viscosities and ratios of flow rates in steady-state flow $R_z = 0.67R_2$), the HDS section has smaller dimensions than in the first case. Here the amount of the pressure drop in the second case is also less, which is connected with lower energy losses in rebuilding the two-layer flow velocity profile.

Thus, conditions at the inlet to the zone of combined flow mainly determine the nature of two-layer flow in the initial section.

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